Theory of Computation: Midterm

1. (20%+20%) Find deterministic finite state automata for the following languages defined on alphabet $\{0, 1\}$.
   (a) (10%) the set of strings containing 00;
   (b) (10%) the set of strings not containing 000;
   (c) (20% *bonus*) the set of strings containing 00 but not 000. (hint: combine the above finite automata)

2. (20%) Use pumping lemma to prove that

$$L = \{w \in \{a, b\}^* : n_a(w) = n_b(w)\},$$

   is not a regular language. (hint: use the string $a^m b^m$)

3. (20%) Given context-free grammar $G = \{\{S\}, \{0, 1\}, S, P\}$, where P is

$$S \rightarrow \lambda \mid 0 \mid 1 \mid 0S0 \mid 1S1,$$

   show that
   (a) (10%) If $w = w^R$, then $w \in L(G)$. (hint: induction on $|w|$)
   (b) (10%) If $w \in L(G)$, then $w = w^R$. (hint: induction on the number of steps in $S \overset{*}{\Rightarrow} w$)

4. (20%) Find an equivalent grammar without λ-production for the following grammar

$$\begin{cases} S \rightarrow ABC \\ A \rightarrow aA \mid \lambda \\ B \rightarrow bB \mid \lambda \\ C \rightarrow cC \mid \lambda. \end{cases}$$

5. (20%) For the following language

$$L = \{a^n b^{2n} : n \geq 1\}.$$

   (a) (10%) Find a CFG in Greibach normal form for L.
   (b) (10%) Find a pushdown automaton for L. (hint: use the construction method from grammar to pda)