

1.1.1

Show $|2^n|=2^{n!}$

As the set is an empty set \varnothing . There is only one element in the set.

$$|2^0|=1=2^{0!}$$

As $|S|=1$, the set is an element.

$$|2^1|=2=2^{1!}$$

Assume $|S| \leq n$, $|2^n|=2^{n!}$ is true

Then, as $|S|=n+1$

It has two case: pre n items including $(n+1)$ th item or not.

$$S=\{a_1, a_2, \dots, a_n\}$$

$$(1) S' = S \cup \{a_{n+1}\}, |S'| = n+1 \quad (2) S' = S, |S'| = n$$

$$\text{So we can get } |2^{n+1}| = 2 * 2^{n!} = 2^{n!+1} = 2^{(n+1)!}$$

By induction, we prove it.

1.2.1

$$\text{As } n=0, |u^0| = |\lambda| = 0 = 0! |u|$$

$$\text{As } n=1, |u^1| = |u| = 1 * |u|$$

Assume $n \leq k$, $|u^k| = k! |u|$ is true

Then, as $n=k+1$

We already know $|u \cdot v| = |u| + |v|$

$$|u^{k+1}| = |u^k \cdot u| = |u^k| + |u| = k! |u| + |u| = (k+1)! |u|$$

By induction, we prove it.

1.3.4

<id>-><letter><rest>

<rest>-><letter><rest>|<digit><rest1>

<rest1>-><letter><rest1>|<digit><rest2>| λ

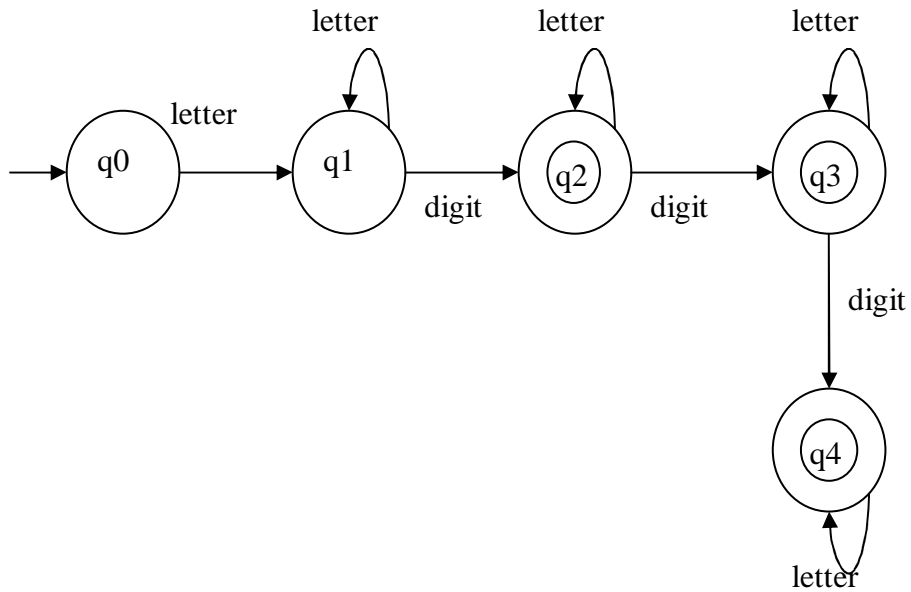
<rest2>-><letter><rest2>|<digit><rest3>| λ

<rest3>-><letter><rest3>| λ

<letter>->a|b|c|...|z|A|B|C...|Z

<digit>->0|1|2|...|9

And an accepter.



From accepter, we can see

$\langle id \rangle \rightarrow \text{letterLdigitL} | \text{letterLdigitLdigitL} | \text{letterLdigitLdigitLdigitL}$
 $L \rightarrow \text{letterL} | \lambda$

2.1.1

0001 and 01001 are accepted.

2.2.1

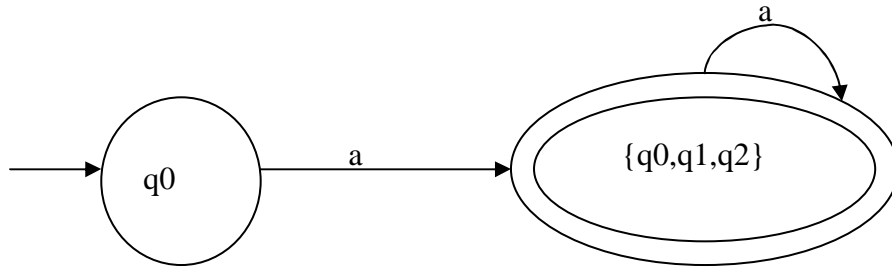
$|w|$ has $(|w|+1)$ interval at most. Every interval has Λ at most.

We get $(|w|+1)\Lambda$.

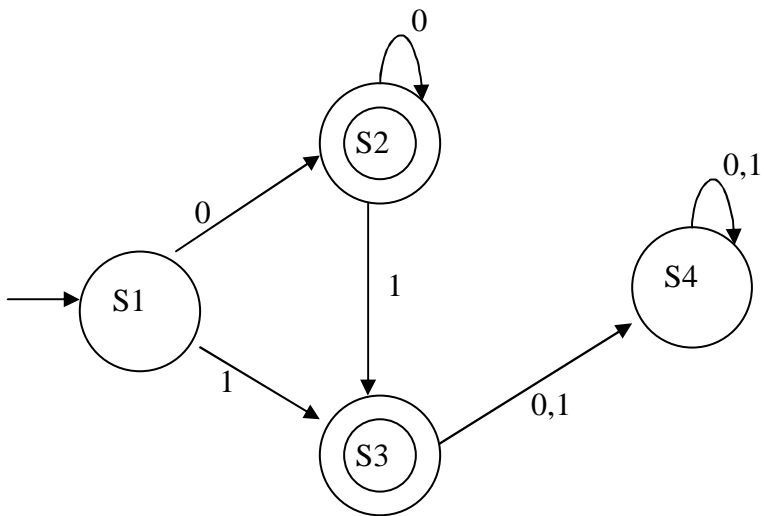
The number of walk is $|w|$.

Total is $(|w|+1)\Lambda + |w| = \Lambda + (\Lambda+1)|w|$

2.3.1



2.4.1



$S1 = \{q0\}$

$S2 = \{q0, q1\} \{q0, q1, q2\}$

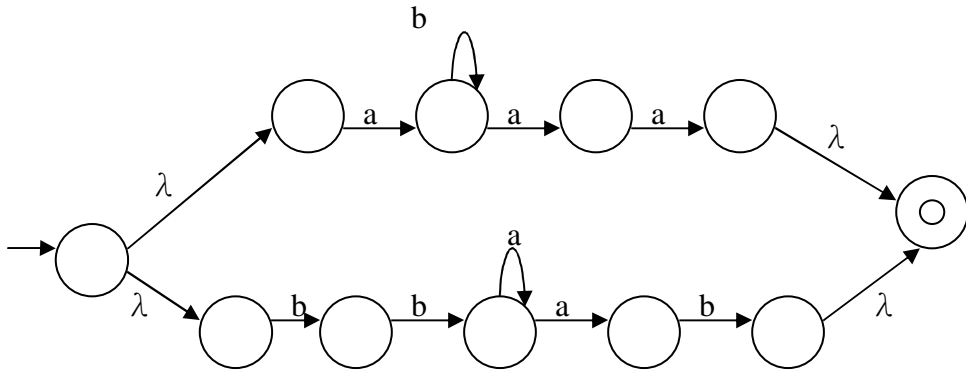
$S3 = \{q1\} \{q1, q2\}$

$S4 = \{q2\} \phi$

3.1.1

b, ab, bb, ba, abb, aba, bba, bab, aab, bbb, baa

3.2.1



3.3.1

