Homework 1: Turn in your work electronically to TA by October 27.

1. Find
$$\begin{bmatrix} 2 & 3 \\ 0 & 0 \end{bmatrix}^n, \quad \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}^n, \quad \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}^{-1}.$$

2. Find the LU decomposition of
$$\begin{bmatrix} 1 & 3 & 5 \\ 3 & 12 & 18 \\ 5 & 18 & 30 \end{bmatrix}.$$

3. Draw the row picture and the column picture for the following system of linear equations
$$\begin{cases} x - 2y = 0 \\ x + y = 6 \end{cases}$$

4. Let
$$\mathbf{A} = \begin{bmatrix} -1 & 1 & 2 \\ 1 & 3 & 0 \end{bmatrix}, \ \mathbf{B} = \begin{bmatrix} 1 & 4 \\ 2 & 2 \\ 3 & 0 \end{bmatrix}$$
Compute $\mathbf{AB}$ and $\mathbf{BA}$ with
   (a) row-by-row computation
   (b) column-by-column computation

5. Use Gauss-Jordan method to find the inverse of
$$\mathbf{A} = \begin{bmatrix} 2 & -1 & 1 \\ 1 & -6 & 5 \\ -3 & 3 & 2 \end{bmatrix}.$$

6. Find an approximate solution to the differential equation
$$-\frac{d^2u(x)}{dx^2} = 4x, \quad 0 \le x \le 1$$
with boundary condition
$$u(0) = 0, \ u(1) = 0$$
at the discrete points of $x = \frac{1}{3}$ and $x = \frac{2}{3}$.

7. Compute the following sums

$$\sum_{i=1}^{5} i, \quad \sum_{\substack{i=2 \\ i \neq 5}}^{6} 2, \quad \sum_{i=2}^{5} i^2(i+1), \quad \sum_{i=1}^{2}\sum_{j=2}^{3}(i-j+1)$$

8. Let

$$\mathbf{E} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \ \mathbf{F} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}, \ \mathbf{G} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Find $\mathbf{E}\,\mathbf{F}\,\mathbf{G}$.

9. Solve the following system of linear equations

$$\begin{cases} x + 3y + 5z = 0 \\ x + 2y + 3z = 3 \\ 3x + 1y + 2z = 2 \end{cases}$$

10. Let matrix $\boldsymbol{S}$ be symmetric and invertible. Show that $\boldsymbol{S}^{-1}$ is symmetric.