Homework 2: Turn in your work electronically to TA by November 9

1. Solve
$$\begin{cases} 1x_1 + 2x_2 + 3x_3 + 5x_4 = 0 \\ 2x_1 + 4x_2 + 8x_3 + 6x_4 = 6 \\ 3x_1 + 6x_2 + 7x_3 + 2x_4 = 2 \end{cases}$$

2. Decide whether the following vectors are linearly independent
$$\begin{bmatrix} 1 \\ 3 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ 1 \\ -1 \end{bmatrix}.$$

3. Consider $\mathbb{R}^3$.
   (a) Find a spanning set of 3 vectors for the plane $x + y + z = 0$.
   (b) Find a basis for the above plane.

4. Find a basis for each fundamental subspace of matrix
$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 & 3 & 5 \\ 2 & 4 & 8 & 12 \\ 3 & 6 & 7 & 13 \end{bmatrix}$$

5. Determine the rank and the fundamental subspaces of matrix
$$\boldsymbol{M} = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 3 & 0 & 3 & -1 \\ 2 & -1 & 2 & -3 \end{bmatrix}$$

6. Consider the plane $\mathcal{P} : x - y - z = 0$ in $\mathbb{R}^3$.
   (a) Express $\mathcal{P}$ as the nullspace of a matrix $\boldsymbol{B}$.
   (b) Find the row space of $\boldsymbol{B}$.

7. What is the echelon matrix of
$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 & 0 & 2 & 1 \\ -1 & -2 & 1 & 1 & 0 \\ 1 & 2 & -3 & -7 & -2 \end{bmatrix}$$

8. Let $\boldsymbol{R} : \mathbb{R}^2 \mapsto \mathbb{R}^2$ be the reflection across the 45° line.

   (a) Find the representation for $\boldsymbol{R}$ with basis $\{v_1 = (1, 0),\ v_2 = (0, 1)\}$

   (b) Find the representation for $\boldsymbol{R}$ with basis $\{V_1 = (1, 1),\ V_2 = (1, -1)\}$

9. (20%) Let $\boldsymbol{T} : \mathbb{M}_{2\times 2} \mapsto \mathbb{M}_{2\times 2}$ maps a 2×2 matrix to its transpose.

   (a) Show that $\boldsymbol{T}$ is linear transformation

   (b) Find a matrix representation for $\boldsymbol{T}$