**Homework 5: Turn in your work electronically to TA by December 31**

1. Find the eigenvalues and the eigenvectors of the following matrix

$$\boldsymbol{T} = \begin{bmatrix} 0.2 & 0.4 & 0.3 \\ 0.4 & 0.2 & 0.3 \\ 0.4 & 0.4 & 0.4 \end{bmatrix}.$$

2. $\boldsymbol{B}$ is a $3 \times 3$ matrix with spectrum $\{1, 2, 3\}$. Find the rank of $\boldsymbol{B}$ and the eigenvalues of $(\boldsymbol{B} + \boldsymbol{I})^{-1}$.

3. Solve the following linear differential equation

$$\frac{d\boldsymbol{u}}{dt} = \boldsymbol{A}\boldsymbol{u}, \text{ where } \boldsymbol{A} = \begin{bmatrix} 2 & 3 \\ 3 & 1 \end{bmatrix} \text{ and } \boldsymbol{u}(0) = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$$

4. If $\boldsymbol{M}^2 = \boldsymbol{M}$, show that $e^{\boldsymbol{M}} = \boldsymbol{I} + (e-1)\boldsymbol{M}$.

5. Diagonalize the following matrices

$$\boldsymbol{C} = \begin{bmatrix} 0 & 1+i \\ 1-i & 1 \end{bmatrix}, \quad \boldsymbol{K} = \begin{bmatrix} 0 & 1-i \\ -1-i & -i \end{bmatrix}$$