Midterm 2021.11.24

1. Find orthonormal vectors $\boldsymbol{q}_1, \boldsymbol{q}_2, \boldsymbol{q}_3$ by the Gram-Schmidt process from

$$\boldsymbol{a}_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \boldsymbol{a}_2 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \boldsymbol{a}_3 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

2. Find LU-decomposition for

$$\boldsymbol{M} = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 5 & 2 \\ -2 & 3 & 2 \end{bmatrix}$$

3. Find QR-factorization for

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

4. Find the projection matrix to the column space of

$$\boldsymbol{B} = \begin{bmatrix} 1 & 1 \\ -2 & -1 \\ -1 & 2 \end{bmatrix}$$

5. Define elementary matrices

$$\boldsymbol{E} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \boldsymbol{F} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}, \boldsymbol{G} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

   (a) Find $\boldsymbol{E}^{13}\,\boldsymbol{F}^{26}\,\boldsymbol{G}^{39}$
   (b) Find $\boldsymbol{G}^{13}\,\boldsymbol{F}^{26}\,\boldsymbol{E}^{39}$

6. Find a basis for the plane $x - 2y + z = 0$ in $\mathbb{R}^3$.

7. Find the matrix for the composite transformation consisting of counter-clockwise rotation of 30° followed by projection on the 45° diagonal line.

8. Fit dataset $\{(-1, 2), (0, 0), (1, -3), (2, -5)\}$ to $y = at^2 + bt + c$.

9. Consider the space $\mathcal{S}$ of all vectors in $\mathbb{R}^6$ with $x_1 - x_2 = x_3 - x_4 = x_5 - x_6$. Find the dimension of $\mathcal{S}$ and a basis for $\mathcal{S}^\perp$.

10. Find $\boldsymbol{A}_3^{-1}$ and $\boldsymbol{A}_4^{-1}$ where

$$\boldsymbol{A}_3 = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}, \boldsymbol{A}_4 = \begin{bmatrix} 1 & -1 & 1 & -1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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