Midterm on 2026 April 15

- Usage of electronic device is forbidden.
- Answers without due explanation/reasoning will not be graded.
- Basic distributions

$$X \sim \textbf{Ber}(p) \;\Rightarrow\; p_X(x) = p^x(1-p)^{1-x} \text{ for } x = 0, 1$$

$$X \sim \textbf{Bin}(n, p) \;\Rightarrow\; p_X(x) = \binom{n}{x} p^x(1-p)^{n-x} \text{ for } x = 0, 1, \ldots, n$$

$$X \sim \textbf{Geo}(p) \;\Rightarrow\; p_X(x) = p(1-p)^{x-1} \text{ for } x = 1, 2, \ldots$$

$$X \sim \textbf{Uni}[a, b] \;\Rightarrow\; p_X(x) = \frac{1}{b-a+1} \text{ for } x = a, a+1, \ldots, b$$

$$X \sim \textbf{Poi}(\lambda) \;\Rightarrow\; p_X(x) = e^{-\lambda}\frac{\lambda^x}{x!} \text{ for } x = 0, 1, \ldots$$

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1. Consider 1D symmetric random walk moving to left or right with equal probability. Starting from the origin, let Z be the number of times point 5 is visited before the walk returns to the origin. Find the variance of Z.

2. Consider a ring of 12 points indexed by $1, \ldots, 12$ in clockwise order (as in a clock). A random walk starts with Point 6 and ends with the first revisit of Point 6. During a walk, the walker moves 1 step clockwise with probability 0.5 or 1 step counter-clockwise with probability 0.5. Let $D = \max_n |X_n - 6|$, where $X_n \in \{1, \ldots, 12\}$ is a point during the walk. Find the variance of D.

3. Jo and Mo both pick 30 days from Year 2027 to go fishing, with all possibilities equally likely. Find the expected number of days that they both pick.

4. Let $X \sim \textbf{Bin}(8, 0.5)$ and $Y \sim \textbf{Bin}(7, 0.5)$ be independent. Find $P(X \leq Y)$.

5. Py and Sy answers questions in turn, with each question answered correctly with probability p. What is the probability that Py, who answers first, answers correctly first?

6. Let M be the number of points played beyond the first deuce until the game is finished. Assuming the server wins a point with probability 0.6, find $E[M]$ and var(M).

7. Consider a fair coin and a biased coin, which lands Heads with probability $3/5$. Suppose a coin is selected (with equal probability) and flipped twice, and the outcome is a Head followed by a Tail. What is the probability that the selected coin is fair? What is the probability that the next flip of this coin is a Head?

8. Find the following probabilities.

   (a) At least one 6 in six throws of a fair dice.

   (b) At least two 6s in twelve throws of a fair dice.

9. Flip a fair coin until the first Head. Let X be the number of Tails before the first Head. What are $E[X]$ and var(X)?

10. Consider a random experiment with a fair dice. What is the expected number of throws until back-to-back 5s occur for the first time?