Homework 4

- Due by 2025.04.30 9:11AM
- Answers without due explanation/reasoning will not be graded.

1. Let M be the number of matches to play until the winner of a best-of-seven series is decided. Assume every match is 50-50. Compute $\mathbf{E}M$ and var M. (hint: find the distribution of M first)

2. Let $X \sim$ **Bin**$(10, 0.6)$ and $Y \sim$ **Bin**$(20, 0.3)$ be independent, and $Z = X + Y$. Compute var Z and cov(X, Z).

3. Consider a ring of 12 points indexed by $1, \ldots, 12$ in clockwise order (as in a clock). A random walk starts with Point 12 and ends with the first revisit back to Point 12. During a walk, the walker moves 1 step clockwise with probability 0.5 or 1 step counter-clockwise with probability 0.5. Let K be the number of visits to Point 5.
   - Compute $p_K(0)$ (hint: gambler's ruin, total probability)
   - Compute $\mathbf{E}K$ and var K (hint: geometric, conditional expectation)

4. Let N be the number of different birthdays for 25 classmates, where each birthday is random uniformly.
   - Find $\mathbf{E}N$ (hint: linearity of expectation)
   - Approximate $p_N(25)$ (hint: the rule of rare events)

5. In a fantasy probability class, the number of register students is Poisson $N \sim$ Poi(60). Each student passes with probability 0.8 or fails with probability 0.2, independent of each other. Define $Z = X - Y$ where X is be the number of students that pass and Y is the number of students that fail. Compute var Z and cov(X, Z). (hint: chicken-egg problem)