## Homework 6

- Due by 2025.05.14 9:11AM
- Answers without due explanation/reasoning will not be graded.
- 1. Let  $X \sim \mathcal{N}(\mu, \sigma^2)$ . Find the second, third, and fourth moments of X. (hint: MGF)
- 2. Find the integral

$$\int_{-\infty}^{\infty} e^{-x^2} dx$$

- 3. Find the MGFs of  $X \sim \text{Ber}(p)$  and  $Y \sim \text{Bin}(n, p)$  and  $G \sim \text{Geo}(p)$ .
- 4. Let  $X \sim \mathcal{N}(0, 1)$  and  $Y = e^X$ . Find  $\mathbf{E} Y^n$ .
- 5. Suppose X and Y are independent normal random variables with  $X \sim \mathcal{N}(\mu_x, \sigma_x^2)$ and  $Y \sim \mathcal{N}(\mu_y, \sigma_y^2)$ . Find the PDF of Z = X + Y. (hint: MGF)
- 6. Let  $X \sim \text{Exp}(1)$  and Y = aX + b where a > 0. Find the PDF of Y. (hint: 2-step method)
- 7. Suppose X has the following 2-sided exponential PDF

$$f_X(x) = \frac{1}{2} e^{-|x|}, \quad x \in \mathbb{R}$$

Find  $\operatorname{var} X$ .