Homework 3

1. Consider 1D symmetric random walk moving to left or right with equal probability. Starting from the origin, let N be the largest absolute distance from the origin before the walk returns to the origin for the first time. Find the PMF of N.

2. Consider a ring of 12 points indexed by $1, \dots, 12$ in clockwise order (as in a clock). A random walk starts with Point 6 and ends with the first revisit of Point 6. During a walk, the walker moves 1 step clockwise with probability 0.5 or 1 step counter-clockwise with probability 0.5. Let $D = \max_n |X_n - 6|$, where $X_n \in \{1, \dots, 12\}$ is a point during the walk. Find the expectation of D.

3. A slot machine outputs a random toy from 5 types, with equal probability for each type, for each inserted coin. To collect all types of toys, what is the expected number of coins to insert?

4. For a fair coin, what is the expected number of tosses until back-to-back heads occur for the first time?

5. Consider a class of 10 girls and 10 boys. Assume birthdays are uniformly distributed over 365 calendar days for the students. Estimate the probability that exactly one boy-girl pair have adjacent birthdays

   - by the Poisson distribution for rare events
   - by simulation (specify the number of trials and the relative frequency)