Quiz 2

1. A chicken lays $N \sim \textbf{Poi}(\lambda)$ eggs, each of which hatches a chick with probability p, independent of other eggs. Let X be the number of eggs that hatch, then we have $X|N = n \sim \textbf{Bin}(n, p)$. Find the marginal PMF p_X and the conditional PMF $p_{N|X}$.

2. Let $X \sim \textbf{Uni}(0, 1)$ and $Y|X = x \sim \textbf{Uni}(0, x)$. Find the marginal PDF f_Y and the conditional PDF $f_{X|Y}$.

3. Break interval $(0, 1)$ into two segments $(0, B)$ and $(B, 1)$, where $B \sim \textbf{Uni}(0, 1)$. Find the covariance and correlation of the lengths of the two segments.

4. Let $X \sim \textbf{Exp}(1)$ and $Y \sim \textbf{Exp}(1)$ be independent. Find the CDF and PDF of

$$S = X + Y$$

5. Let $X \sim \textbf{Uni}(0, 1)$ and $Y \sim \textbf{Uni}(0, 1)$ be independent. Find the CDF and PDF of

$$R = \frac{X}{Y}$$