Since the statistical distribution of an HMM also includes the distribution of observations given the Markov state, which is called the emission distribution, this model is called a hidden Markov model (HMM).

The following relation holds for the time-invariant state space.

\[ \beta_t = \sum_{i=1}^{S} p_{it} \beta_{i,t+1} \]

Example: Coin Toss

Suppose that there are two heads and two tails. Then \( S = 2 \) and \( A = 1 \), which is the up state, and \( o = \{1, 1, 0, 0, 0\} \) is the observed sequence. The HMM can be expressed as follows:

\[ p_a = \frac{1}{2} \]

\[ p_{ij} = \begin{cases} \frac{1}{2}, & i = j \leq 1 \\ 0, & i = j > 1 \\ \frac{1}{2}, & i = 1, j = 0 \\ \frac{1}{2}, & i = 0, j = 1 \end{cases} \]

\[ b_1 = \frac{1}{2}, b_0 = 0 \]

\[ \lambda = (A, P, B) \]

\[ \alpha_t \]

\[ \beta_t \]

\[ s_t \]

\[ o_t \]

\[ n \]

The following relation holds for the time-invariant state space.

\[ \beta_t = \sum_{i=1}^{S} p_{it} \beta_{i,t+1} \]

Estimation Problem

Find the maximum likelihood estimate of the model parameters, given the observed sequence. The maximum likelihood estimate of the model parameters is obtained by maximizing the probability of the observed sequence.

Evaluation Problem

Find the expected number of heads and tails in the observed sequence.

The expected number of heads and tails in the observed sequence is given by

\[ E(h) = \sum_{t=1}^{n} o_t \]

\[ E(t) = n - E(h) \]

Example of Markov Chain

Let \( S = 2 \) and \( A = 1 \), which is the up state, and \( o = \{1, 1, 0, 0, 0\} \) is the observed sequence. The HMM can be expressed as follows:

\[ p_a = \frac{1}{2} \]

\[ p_{ij} = \begin{cases} \frac{1}{2}, & i = j \leq 1 \\ 0, & i = j > 1 \\ \frac{1}{2}, & i = 1, j = 0 \\ \frac{1}{2}, & i = 0, j = 1 \end{cases} \]

\[ b_1 = \frac{1}{2}, b_0 = 0 \]

\[ \lambda = (A, P, B) \]

\[ \alpha_t \]

\[ \beta_t \]

\[ s_t \]

\[ o_t \]

\[ n \]

The following relation holds for the time-invariant state space.

\[ \beta_t = \sum_{i=1}^{S} p_{it} \beta_{i,t+1} \]

Estimation Problem

Find the maximum likelihood estimate of the model parameters, given the observed sequence. The maximum likelihood estimate of the model parameters is obtained by maximizing the probability of the observed sequence.

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